Name _____ Date _____

Gaussian Elimination

Example 1... Solve the following system of equations BY HAND using substitution.

a. $\begin{cases} x - 2y + z = -4 \\ -3y + z = -7 \\ z = -4 \end{cases}$

$-3y - 4 = -7$

$-3y = -3$

$y = 1$

$(2, 1, -4)$

$x - 2(1) - 4 = -4$

$x - 2 - 4 = -4$

$x - 6 = -4 \quad x = 2$

Gaussian elimination is another method for solving a system of linear equations with 3 variables. You want your system to get to triangular form which looks like: $\begin{cases} x + 2y + z = 0 \\ y - 6z = 6 \\ z = -1 \end{cases}$.

Notice the first equation has all three variables, the second only has y and z and the third only z. And each of the leading coefficients on x, y and z are 1 (we will try to make this happen as best we can, but sometimes it won't happen). This is what we want our equations to look like because the problem becomes an easy substitution problem like our first example. We can: 1) swap two rows
2) multiply a row by a non-zero number
3) add a multiply of one row to another row

Example 2... Solve the following systems of equations BY HAND using Gaussian elimination.

a. $\begin{cases} 4x - y + z = 3 \\ x + 2y + z = 0 \\ 3x + 7y - 3z = 6 \end{cases}$

$\begin{cases} x + 2y + z = 0 \\ 4x - y + z = 3 \\ 3x + 7y - 3z = 6 \end{cases}$

$\begin{cases} x + 2y + z = 0 \\ -9y - 3z = 3 \\ 3x + 7y - 3z = 6 \end{cases}$

$\begin{cases} x + 2y + z = 0 \\ -9y - 3z = 3 \\ y - 6z = 6 \end{cases}$

$\begin{cases} x + 2y + z = 0 \\ y - 6z = 6 \\ -9y - 3z = 3 \end{cases}$

$\begin{cases} x + 2y + z = 0 \\ y - 6z = 6 \\ z = -1 \end{cases}$

$y - 6(-1) = 6$

$y + 6 = 6$

$y = 0$

$x + 2(0) - 1 = 0$

$x + 0 - 1 = 0$

$x = 1$

$(1, 0, -1)$

$R1 \leftrightarrow R2$

$-4R1 + R2 = R2$

$-4x - 8y - 4z = 0$

$+ 4x - y + z = 3$

$-9y - 3z = 3$

$-3R1 + R3 = R3$

$-3x - 6y - 3z = 0$

$+ 3x + 7y - 3z = 6$

$y - 6z = 6$

$R2 \leftrightarrow R3$

$9R2 + R3 = R3$

$9y - 54z = 54$

$+ -9y - 3z = 3$

$-57z = 57$

$z = -1$

b. $\begin{cases} x-3y+z=4 \\ 2x-8y+8z=-2 \\ -6x+3y-15z=9 \end{cases}$

$\begin{cases} x-3y+z=4 \\ -2y+6z=-10 \\ -6x+3y-15z=9 \end{cases}$

$\begin{cases} x-3y+z=4 \\ y-3z=5 \\ -6x+3y-15z=9 \end{cases}$

$\begin{cases} x-3y+z=4 \\ y-3z=5 \\ -15y-9z=33 \end{cases}$

$\begin{cases} x-3y+z=4 \\ y-3z=5 \\ z=-2 \end{cases}$

$-2R1+R2=R2$

$-2x+6y-2z=-8$

$2x-8y+8z=-2$

$-2y+6z=-10$

$R2 \div -2$

$6R1+R3=R3$

$6x-18y+6z=24$

$-6x+3y-15z=9$

$-15y-9z=33$

$y-3(-2)=5$

$y+6=5$

$y=-1$

$x-3(-1)-2=4$

$x+3-2=4$

$x+1=4 \quad x=3$

$(3,-1,-2)$

$15R2+R3=R3$

$15y-45z=75$

$+\ -15y-9z=33$

$-54z=108$

$z=-2$

c. $\begin{cases} 2x+y-z=5 \\ 3x-y+2z=-1 \\ x-y-z=0 \end{cases}$

$\begin{cases} x-y-z=0 \\ 3x-y+2z=-1 \\ 2x+y-z=5 \end{cases}$

$\begin{cases} x-y-z=0 \\ 2y+5z=-1 \\ 2x+y-z=5 \end{cases}$

$\begin{cases} x-y-z=0 \\ 2y+5z=-1 \\ 3y+z=5 \end{cases}$

$\begin{cases} x-y-z=0 \\ 2y+5z=-1 \\ z=-1 \end{cases}$

$2y+5(-1)=-1$

$2y-5=-1$

$2y=4$

$y=2$

$x-2--1=0$

$x-2+1=0$

$x-1=0 \quad x=1$

$(1,2,-1)$

$R1 \leftrightarrow R3$

$-3R1+R2=R2$

$-3x+3y+3z=0$

$3x-y+2z=-1$

$2y+5z=-1$

$-2R1+R3=R3$

$-2x+2y+2z=0$

$+\ 2x+y-z=5$

$3y+z=5$

$3R2+-2R3=R3$

$6y+15z=-3$

$+\ -6y-2z=-10$

$13z=-13 \quad z=-1$

d. $\begin{cases} x+2y-5z=-11 \\ 4x-3y+2z=11 \\ -3x+5y+3z=8 \end{cases}$

$\begin{cases} x+2y-5z=-11 \\ -11y+22z=55 \\ -3x+5y+3z=8 \end{cases}$

$\begin{cases} x+2y-5z=-11 \\ y-2z=-5 \\ -3x+5y+3z=8 \end{cases}$

$\begin{cases} x+2y-5z=-11 \\ y-2z=-5 \\ 11y-12z=-25 \end{cases}$

$\begin{cases} x+2y-5z=-11 \\ y-2z=-5 \\ z=3 \end{cases}$

$y-2(3)=-5$

$y-6=-5$

$y=1$

$x+2(1)-5(3)=-11$

$x+2-15=-11$

$x-13=-11$

$x=2$

$(2,1,3)$

$-4R1+R2=R2$

$-4x-8y+20z=44$

$+\ 4x-3y+2z=11$

$-11y+22z=55$

$R2 \div -11$

$3R1+R3=R3$

$3x+6y-15z=-33$

$+\ -3x+5y+3z=8$

$11y-12z=-25$

$-11R2+R3=R3$

$-11y+22z=55$

$+\ 11y-12z=-25$

$10z=30 \quad z=3$