

Absolute Value Transformations Notes

Given a parent function $f(x)$, the transformation of the function is given by: $f(x) = a|b(x - h)| + k$

- Each of the letters a , b , h , and k have a job.

a : vertical stretch, compression and/or reflection, affects y

b : horizontal stretch, compression and/or reflection, affects x (use the reciprocal)

h : horizontal shift left or right, affects x (use the opposite)

k : vertical shift up or down, affects y

Example 1... Describe each transformation.

a. $g(x) = |x + 4| - 6$

horizontal shift left 4
vertical shift down 6

b. $h(x) = -3|x - 7| + 1$

vert. reflection
vert. stretch of 3
horiz. shift right 7
vert. shift up 1

c. $f(x) = |3x - 2| = |3(x - \frac{2}{3})|$

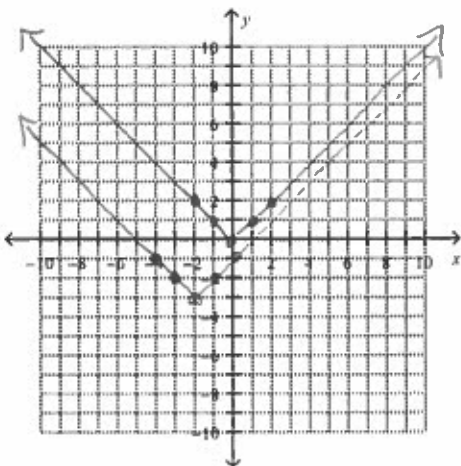
horiz. compression of 3
horiz. shift right $\frac{2}{3}$

d. $h(x) = \frac{1}{3}|2(x + 1)| - 3$

vert. compression of $\frac{1}{3}$
horiz. compression of 2
horiz. shift left 1
vert. shift down 3

Example 2... Using the graph of $f(x) = |x|$ and the table, graph the transformation:

$f(x + 2) - 3$.



opposite! ↓

$y = |x|$

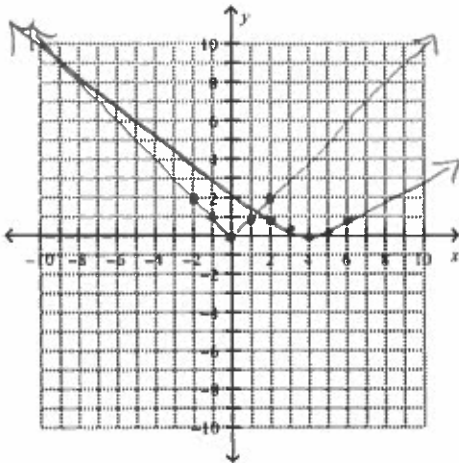
$x - 2$	x	y	$y - 3$
-4	-2	2	-1
-3	-1	1	-2
-2	0	0	-3
-1	1	1	-2
0	2	2	-1

Example 3... Use tables to graph the transformations of the absolute value parent function. Identify the vertex and the domain and range of each transformed function.

a. $\frac{1}{3}f(x - 4)$

$x+4$	x	y	$\frac{1}{3}y$
2	-2	2	$\frac{2}{3}$
3	-1	1	$\frac{1}{3}$
4	0	0	0
5	1	1	$\frac{1}{3}$
6	2	2	$\frac{2}{3}$

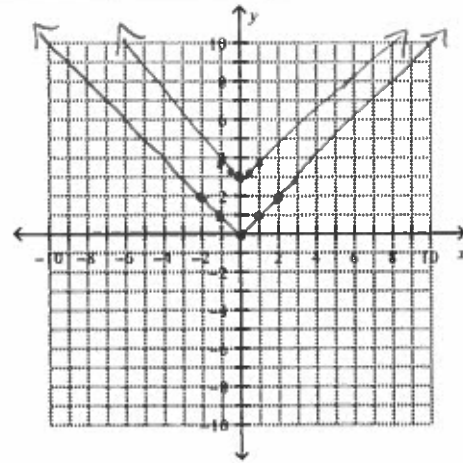
$V = (4, 0)$
 $D: (-\infty, \infty)$
 $R: [0, \infty)$



b. $\frac{1}{3}f(-2x) + 3$

$-\frac{1}{2}x$	x	y	$\frac{1}{3}y + 3$
1	-2	2	$3\frac{2}{3}$
$\frac{1}{2}$	-1	1	$3\frac{1}{3}$
0	0	0	3
$-\frac{1}{2}$	1	1	$3\frac{1}{3}$
-1	2	2	$3\frac{2}{3}$

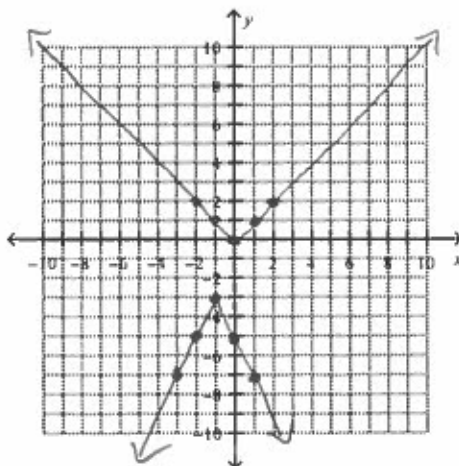
$V = (0, 3)$
 $D: (-\infty, \infty)$
 $R: [3, \infty)$



c. $f(x) = -2|x + 1| - 3$

$x-1$	x	y	$-2y-3$
-3	-2	2	-7
-2	-1	1	-5
-1	0	0	-3
0	1	1	-5
1	2	2	-7

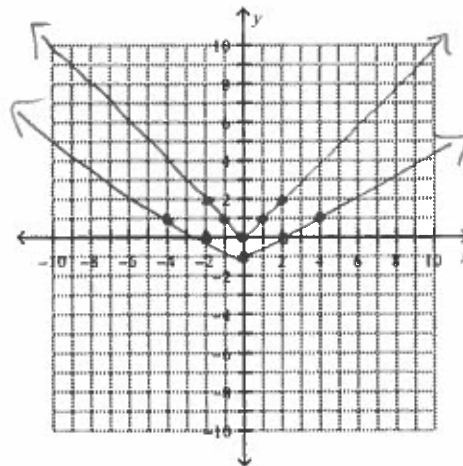
$V = (-1, -3)$
 $D: (-\infty, \infty)$
 $R: (-\infty, -3]$



d. $h(x) = \frac{1}{2}|x| - 1$

$2x$	x	y	$y-1$
2	-2	2	1
-2	-1	1	0
0	0	0	-1
2	1	1	0
4	2	2	1

$V = (0, -1)$
 $D: (-\infty, \infty)$
 $R: [-1, \infty)$

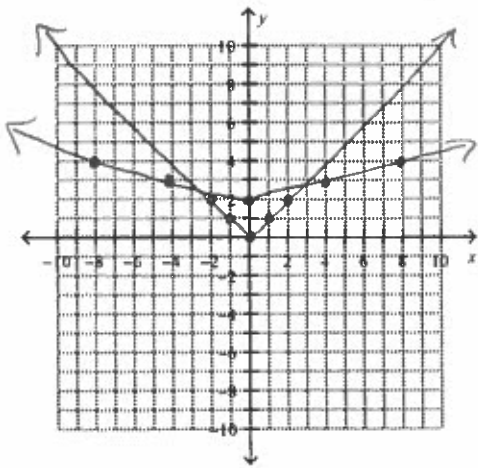


Example 4... Write a transformed absolute value function in terms of the basic absolute value function $f(x) = |x|$ for the given characteristics. Then, graph the transformed function.

a. Horizontal stretch of $\frac{1}{4}$, Vertical shift up 2

$$y = |\frac{1}{4}x| + 2$$

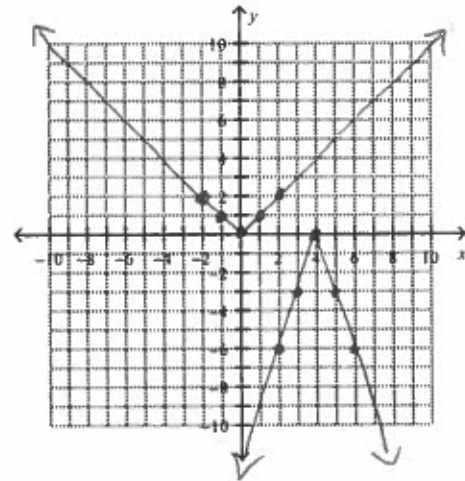
$4x$	x	y	$y+2$
-8	-2	2	4
-4	-1	1	3
0	0	0	2
4	1	1	3
8	2	2	4



b. Reflected across the x-axis, Vertical stretch of 3
Horizontal shift right 4

$$y = -3|x-4|$$

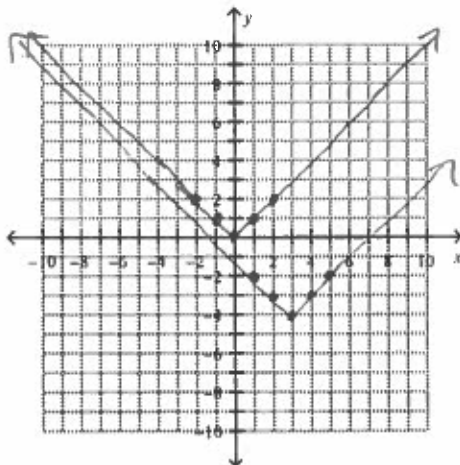
$x+4$	x	y	$-3y$
2	-2	2	-6
3	-1	1	-3
4	0	0	0
5	1	1	-3
6	2	2	-6



a. Vertex (3, -4), Range $[-4, \infty)$

$$y = |x-3| - 4$$

$x+3$	x	y	$y-4$
1	-2	2	-2
2	-1	1	-3
3	0	0	-4
4	1	1	-3
5	2	2	-2



b. Axis of symmetry $x = -1$, Range $(-\infty, 2]$

Vertical compression of $\frac{1}{4}$ $y = -\frac{1}{4}|x+1| + 2$

$x-1$	x	y	$-\frac{1}{4}y+2$
-3	-2	2	$1\frac{1}{2}$
-2	-1	1	$1\frac{3}{4}$
-1	0	0	2
0	1	1	$1\frac{3}{4}$
1	2	2	$1\frac{1}{2}$

