Name ____________________ Date ____________

Exponential Growth & Decay Notes

Exponential functions are functions that have a variable (x) in the exponent.

Standard form for exponential functions : $y = ab^x$

a = starting value (initial)

b = growth/decay factor

If $b > 1$, growth

If $0 < b < 1$, decay

Example 1... Without graphing, determine whether each function represents exponential growth or decay. Also determine the percent increase or decrease.

a. $y = 100(0.12)^x$

Decay

$1 - .12 = .88$

88% decrease

b. $y = 0.2(1.74)^x$

Growth

$1.74 - 1 = .74$

74% increase

c. $y = 16(3.4)^x$

Growth

$3.4 - 1 = 2.4$

240% increase

d. $f(x) = 32\left(\frac{14}{10}\right)^x$

Growth

$\frac{14}{10} = 1.4 - 1 = .4$

40% increase

Example 2... A new car that sells for $18,000 depreciates 25% each year. Write a function that models the value of the car. Find the value of the car after 4 years.

$a = 18000$

$b = 1 - .25 = .75$

$x = 4$

$y = ?$

$y = 18000(.75)^x$

$y = 18000(.75)^4$

$y = 18000(.3164)$

$y = \$5,695.31$

Example 3... An abandoned house has a mouse population of 22. It is increasing at a rate of 5% per month. Write a function that models the population. Estimate when there will be 50 mice in the house.

$a = 22$

$b = 1 + .05 = 1.05$

$x = ?$

$y = 50$

$y = 22(1.05)^x$

$50 = 22(1.05)^x$

$2.273 = 1.05^x$

$x = 16$ to 17

Between 16 + 17 months

Example 4... Write an exponential function $y = ab^x$ for a graph that includes the points:

a. (0, 2) and (1, 6)

x y x y

$2 = ab^0$

$2 = a$

$6 = 2b^1$

$\frac{6}{2} = \frac{2b}{2}$

$b = 3$

$\boxed{y = 2(3)^x}$

b. (2, 4) and (3, 16)

x y x y

$\frac{4}{b^2} = \frac{ab^2}{b^2}$

$a = \frac{4}{b^2}$

$16 = \frac{4}{b^2} \cdot b^3$

$\frac{16}{4} = \frac{4b}{4}$

$b = 4 \to$

$a = \frac{4}{4^2} = \frac{4}{16} = \frac{1}{4}$

$\boxed{y = \frac{1}{4}(4)^x}$

Example 5... For each annual rate of change, find the corresponding growth or decay factor.

a. $+35\%$

$b = 1 + .35 = 1.35$

b. -47%

$b = 1 - .47 = .53$

Exponential functions are used to represent many different real-life situations. Here are a few:

The Richter Scale!

The energy released by an earthquake can be represented by the equation: $E \cdot 30^x$, where x is the magnitude of the earthquake on the Richter Scale. (E represents some amount of energy that was present in the earth "originally" – it's not important for this application.)

$\frac{E \cdot 30^x}{E \cdot 30^x}$

Example 1... In 1995, an earthquake in Mexico registered 8.0 on the Richter scale. In 2001, an earthquake of magnitude 6.8 shook Washington state. Compare the amounts of energy released in the two earthquakes.

$\frac{\cancel{E} \cdot 30^8}{\cancel{E} \cdot 30^{6.8}} = \frac{30^8}{30^{6.8}} = 30^{1.2} = 59.23$

The Mexico earthquake was 59.23 times stronger than Wash.

Half-life!

Half-life is a method of figuring out how old certain substances are. When we know how quickly a substance decays, we can use that information to find out how long it's been sitting around.

$y = a\left(\frac{1}{2}\right)^{\frac{t}{k}}$

a = initial amount

t = time

k = half-life

Example 2...

a) Technetium-99m has a half-life of 6 hours. Find the amount of technetium-99m that remains from a 50-mg supply after 25 hours.

$a = 50$

$t = 25$

$k = 6$

$y = 50\left(\frac{1}{2}\right)^{\frac{25}{6}}$

$y = 50\left(\frac{1}{2}\right)^{4.167}$

$y = 50(.5568) =$ 2.78 mg remaining

b) Arsenic-74 is used to locate brain tumors. It has a half-life of 17.5 days. Write an exponential decay function for a 90-mg sample. Use the function to find the amount remaining after 6 days.

$a = 90$
$t = 6$
$k = 17.5$

$y = 90\left(\frac{1}{2}\right)^{\frac{6}{17.5}}$
$y = 90\left(\frac{1}{2}\right)^{.343}$
$y = 90(.7885) = 70.96$ mg remaining

Compounded Interest!

When you put money in an investment account, or even just a bank, it collects interest. This means that the bank will give you a small amount of money just for letting your money sit in their bank. Interest is "compounded", or calculated, a certain number of times per year.

$y = a\left(1+\frac{r}{n}\right)^{nt}$

a = initial amount
n = # times compounded
r = interest rate
t = time

Example 3... **You invest $100 at an annual interest rate of 4%, compounded *quarterly*. How much money will you have in the account after 25 years?**

$a = 100$
$n = 4$
$r = .04$
$t = 25$

$y = 100\left(1+\frac{.04}{4}\right)^{4(25)}$
$y = 100(1.01)^{100}$
$y = 100(2.705) = \$270.48$

Continuously compounded interest is what it sounds like – there are not a certain number of times the interest is calculated – it's done continuously, or all the time. There's a separate formula for that situation:

$A = Pe^{rt}$

P = initial amount
r = interest rate
t = time

Example 3...

a) You invest $1050 at an annual interest rate of 5.5% compounded continuously. How much money will you have in the account after ten years?

$P = 1050$
$r = .055$
$t = 10$

$A = 1050e^{.055(10)}$
$A = 1050e^{.55}$
$A = 1050(1.733)$
$A = \$1,819.92$

b) A student wants to save $8000 for college in five years. How much should be put into an account that earns 5.2% annual interest compounded continuously?

$P = ?$
$r = .052$
$t = 5$
$A = 8000$

$8000 = Pe^{.052(5)}$
$8000 = Pe^{.26}$
$\frac{8000}{1.297} = \frac{P(1.297)}{1.297}$

$P = \$6,168.41$