

Exponential and Log Functions

Notes: Lesson 2

A logarithmic (log) function is the INVERSE of an exponential function. In other words, if you had a variable in the exponent and were trying to solve for that variable, you may use a log function in the solving process.

We can use the following rule to convert between a log function and an exponential function:

$$\log_b m = n \iff b^n = m$$

Example 1... Write each equation in logarithmic form.

a. $5^2 = 25$
 $b \quad m$

$$\log_5 25 = 2$$

b. $10^3 = 1000$
 $b \quad m$

$$\log_{10} 1000 = 3$$

$$\log 1000 = 3$$

c. $729 = 3^{6n}$
 $b \quad m$

$$\log_3 729 = 6$$

d. $7.389 = e^{2n}$
 $b \quad m$

$$\log_e 7.389 = 2$$

$$\downarrow$$

$$\ln 7.389 = 2$$

Example 2... Write each equation in exponential form.

a. $\log_2 8 = 3$
 $b \quad m \quad n$

$$2^3 = 8 \quad (\text{true})$$

b. $\log 100 = 2$

$$\log_{10} 100 = 2$$

$$10^2 = 100$$

c. $\ln 5 = 1.609$

$$\log_e 5 = 1.609$$

$$e^{1.609} = 5$$

d. $\log_5 625 = 4$
 $b \quad m \quad n$

$$5^4 = 625$$

Example 3... Evaluate each logarithm. → set equal to x

a. $\log_4 16 = x$

$$4^x = 16$$

$$x = 2$$

b. $\log_3 \frac{1}{9} = x$

$$3^x = \frac{1}{9}$$

$$x = -2$$

c. $\log_9 27 = x$

$$9^x = 27$$

$$(3^2)^x = 3^3 \quad 2x = 3$$

$$3^{2x} = 3^3 \quad x = \frac{3}{2}$$

d. $\ln e^6$

$$\log_e e^6 = x$$

$$e^x = e^6$$

$$x = 6$$

Example 4... Find the inverse of each logarithmic function.

a. $y = \log_5 x$

$$x = \log_5 y$$

$$5^x = y$$

$$y = 5^x$$

b. $y = \log_2 (x + 3)$

$$x = \log_2 (y + 3)$$

$$2^x = y + 3$$

$$-3 \quad -3$$

$$y = 2^x - 3$$

c. $y = \ln x + 7$

$$x = \ln y + 7$$

$$-7 \quad -7$$

$$x - 7 = \ln y$$

$$x - 7 = \log_e y$$

$$e^{x-7} = y$$

$$y = e^{x-7}$$