

Properties of Logs

Exponent Rule	Property	Example
$a^m \cdot a^n = a^{m+n}$	Product Property $\log_b M + \log_b N = \log_b MN$	$\log_3 2 + \log_3 X = \log_3 2x$
$\frac{a^m}{a^n} = a^{m-n}$	Quotient Property $\log_b M - \log_b N = \log_b \frac{M}{N}$	$\log_4 X - \log_4 Y = \log_4 \frac{X}{Y}$
$(a^m)^n = a^{m \cdot n}$	Power Property $\log_b M^a = a \log_b M$	$\log_5 x^2 = 2 \log_5 x$

Example 1... Write each logarithmic expression as a single logarithm (condense).

a. $\log 7 + \log 2$

$$\log(7)(2) = \log 14$$

b. $\log_2 12 - \log_2 3$

$$\log_2 \frac{12}{3} = \log_2 4$$

c. $\log_3 8 - 2\log_3 6 + \log_3 3$

$$\log_3 8 - \log_3 6^2 + \log_3 3$$

$$\log_3 \frac{8}{36} + \log_3 3$$

$$\log_3 \left(\frac{2}{9}\right)(3) = \log_3 \frac{2}{3}$$

Example 2... Expand each logarithm.

a. $\log_8 x^3 y^5$

$$\log_8 x^3 + \log_8 y^5$$

$$3\log_8 x + 5\log_8 y$$

b. $\log 8\sqrt{x}$

$$\log 8 + \log \sqrt{x}$$

$$\log 8 + \log x^{1/2}$$

$$\log 8 + \frac{1}{2}\log x$$

c. $\ln(7x)^3$

$$\ln 7^3 x^3$$

$$\ln 343 + \ln x^3 \quad \text{or} \quad \ln 7^3 + \ln x^3$$

$$\ln 343 + 3\ln x \quad 3\ln 7 + 3\ln x$$

d. $\log_m 25x^4$

$$\log_m 25 + \log_m x^4$$

$$\log_m 25 + 4\log_m x$$

Example 3... Use the properties of logs to evaluate each expression.

a. $3\log_2 2 - \log_2 4$

$$\log_2 2^3 - \log_2 4$$

$$\log_2 \frac{8}{4} = \log_2 2$$

Evaluate: $\log_2 2 = x$

$$2^x = 2 \quad x = 1$$

c. $6\ln e$

$$\ln e^6$$

Evaluate: $\ln e^6 = x$

$$\log_e e^6 = x$$

$$e^x = e^6 \quad x = 6$$

b. $\log_3 3 + 5\log_3 3$

$$\log_3 3 + \log_3 3^5$$

$$\log_3 (3 \times 243)$$

$$\log_3 729$$

Evaluate: $\log_3 729 = x$

$$3^x = 729 \quad x = 6$$

Example 4... Assume that $\log 3 \approx 0.4771$, $\log 4 \approx 0.6021$, $\log 5 \approx 0.6990$. Use the properties of logarithms to evaluate each expression. Do NOT use a calculator.

a. $\log 15$

$$\log (3)(5)$$

$$\log 3 + \log 5$$

$$.4771 + .6990$$

$$1.1761$$

b. $\log 16$

$$\log (4)(4)$$

$$\log 4 + \log 4$$

$$.6021 + .6021$$

$$1.2042$$

or $\log 4^2$

$$2 \log 4$$

$$2(.6021)$$

$$1.2042$$

c. $\log \frac{4}{3}$

$$\log 4 - \log 3$$

$$.6021 - .4771$$

$$.125$$