NOTES: Lesson 5

Name _____ Date _____

Solving Exponential and Logarithmic Equations

In order to solve exponential equations, we turn them into log equations. Then, we need the change of base formula:

$$\log_b M = \frac{\log M}{\log b}$$

Example 1... **Use the change of base formula to evaluate each logarithm.**

a. $\log_3 42$

$$\frac{\log 42}{\log 3} = 3.402$$

b. $\log_{12} 8$

$$\frac{\log 8}{\log 12} = .837$$

Example 2... **Solve each exponential equation.** -> change to log form

a. $7^{3x} = 20$

$$\log_7 20 = 3x$$

$$\frac{\log 20}{\log 7} = 3x$$

$$\frac{1.5395}{3} = \frac{3x}{3}$$

$$x = .5132$$

b. $3^{x+4} = 101$

$$\log_3 101 = x+4$$

$$\frac{\log 101}{\log 3} = x+4$$

$$4.2009 = x+4$$

$$-4 \quad -4$$

$$x = .2009$$

c. $10^x = 19$

$$\log_{10} 19 = x$$

$$\frac{\log 19}{\log 10} = x \quad \text{or} \quad \log 19 = x$$

$$x = 1.2788 \quad x = 1.2788$$

d. $e^{2x+1} = 37$

$$\log_e 37 = 2x+1$$

$$\ln 37 = 2x+1$$

$$3.6109 = 2x+1$$

$$\frac{2.6109}{2} = \frac{2x}{2}$$

$$x = 1.3055$$

e. $\frac{5e^{6x+3}}{5} = \frac{0.1}{5}$

$e^{6x+3} = .02$

$\log_e .02 = 6x+3$

$\ln .02 = 6x+3$

$-3.912 = 6x+3$

$x = -1.152$

f. $e^{\ln 5x} = 20$

$\log_e 20 = \ln 5x$

$\ln 20 = \ln 5x$ or $\frac{20}{5} = \frac{5x}{5}$

$\ln 20 = \ln 5 + \ln x$ $x = 4$

$-\ln 5 \quad -\ln 5$

$\ln \frac{20}{5} = \ln x$

$\ln 4 = \ln x$ $x = 4$ ☺

Example 2... **Solve each logarithmic/natural logarithmic equation.**

a. $\log_{10}(3x + 1) = 5$

$10^5 = 3x+1$

$100{,}000 = 3x+1$

$-1 \quad -1$

$\frac{99{,}999}{3} = \frac{3x}{3}$

$x = 33{,}333$

b. $\frac{2 \log x}{2} = \frac{-1}{2}$ → change to exp. form

$\log_{10} x = -\frac{1}{2}$

$10^{-\frac{1}{2}} = x$

$x = .3162$

c. $\log_2 (6x) - 3 = -2$

$+3 \quad +3$

$\log_2 6x = 1$

$\frac{2^1}{6} = \frac{6x}{6}$

$x = \frac{1}{3}$

d. $\ln (15x) = 4$

$\log_e 15x = 4$

$e^4 = 15x$

$\frac{54.5982}{15} = \frac{15x}{15}$

$x = 3.6399$

e. $\ln x + \ln 4 = 2$

$\ln 4x = 2$

$\log_e 4x = 2$

$e^2 = 4x$

$\frac{7.3891}{4} = \frac{4x}{4}$

$x = 1.8473$

f. $\frac{5 \ln(3x-2)}{5} = \frac{15}{5}$

$\ln (3x-2) = 3$

$\log_e (3x-2) = 3$

$e^3 = 3x-2$

$20.0855 = 3x-2$

$+2 \quad +2$

$\frac{22.0855}{3} = \frac{3x}{3}$ $x = 7.3618$