

Log Equations with Word Problems!

The hydrogen ions in a substance determines how acidic it is. The more hydrogen ions, the more acidic. The pH and number of hydrogen ions are related in the formula: $pH = -\log [H^+]$

Example 1... The pH of milk is 6.6, while the pH of seawater is 8.5. Find the concentration of hydrogen ions in each substance. Which substance is more acidic?

$$\frac{6.6}{-1} = \frac{-\log [H^+]}{-1}$$

$$-6.6 = \log_{10} [H^+]$$

$$10^{-6.6} = [H^+] = 2.512 \times 10^{-7}$$

$$\frac{8.5}{-1} = \frac{-\log [H^+]}{-1}$$

$$-8.5 = \log_{10} [H^+]$$

$$10^{-8.5} = [H^+] = 3.162 \times 10^{-9}$$

Milk is more acidic.

Example 2... The U.S. population of peninsular bighorn sheep was 1,170 in 1971. By 1999, only 335 remained.

a. Write an exponential equation to model the decline in population.

$$\begin{matrix} (0, 1170) \\ (28, 335) \end{matrix}$$

$$y = 1170 (.9563)^x$$

b. In what year might only 5 bighorn sheep remain in the United States?

$$5 = 1170 (.9563)^x$$

$$.0043 = .9563^x$$

$$\log .9563 .0043 = x$$

$$x = 121.95 \approx 122$$

$$\begin{array}{r} 1971 \\ + 122 \\ \hline \end{array}$$

In yr. 2093

Example 3... An investment of \$2,000 earns 5.75% interest, which is compounded quarterly.

$$y = a \left(1 + \frac{r}{n}\right)^{nt}$$

a. Write an exponential equation to represent the amount of money in the bank over time.

$$y = 2000 \left(1 + \frac{.0575}{4}\right)^{4t}$$

$$y = 2000 (1.014375)^{4t}$$

b. After approximately how many years will the investment be worth \$3,000?

$$3000 = 2000 (1.014375)^{4t}$$

$$1.5 = 1.014375^{4t}$$

$$\log_{1.014375} 1.5 = 4t$$

$$28.4085 = 4t$$

$$t \approx 7.1 \text{ years}$$

Example 4... A parent raises a child's allowance by 20% each year. The child's allowance now is \$8.

a. Write an exponential equation to represent the child's allowance over time.

$$y = 8(1.2)^x$$

b. When will the child's allowance reach \$20?

$$20 = 8(1.2)^x$$

$$2.5 = 1.2^x$$

$$\log_{1.2} 2.5 = x$$

$$x \approx 5.03 \text{ years}$$

Example 5... An investment of \$100 is now valued at \$149.18. The interest rate is 8%, compounded continuously. About how long has the money been invested?

$$149.18 = 100 e^{.08t}$$

$$1.4918 = e^{.08t}$$

$$\log_e 1.4918 = .08t$$

$$\ln 1.4918 = .08t$$

$$.40399 = .08t$$

$$t \approx 5.05 \text{ years}$$

Example 6... An investor sold 100 shares of stock valued at \$34.50 per share. The stock was purchased at \$7.25 per share two years ago. Find the rate of continuously compounded interest in this investment.

$$34.5 = 7.25 e^{2r}$$

$$4.7586 = e^{2r}$$

$$\log_e 4.7586 = 2r$$

$$\ln 4.7586 = 2r$$

$$1.5599 = 2r$$

$$.7799 = r$$

$$r \approx 78\%$$