

Graphs of Square Root Functions

Notes: **Let's talk** about the idea of an inverse function. An inverse function is kind of like the idea of "backwards operations" – when you solve equations, you are using inverse operations to get the variable by itself.

Solve the equation $x^2 = 16$. $x = \pm 4$

What did you do to get the x by itself?

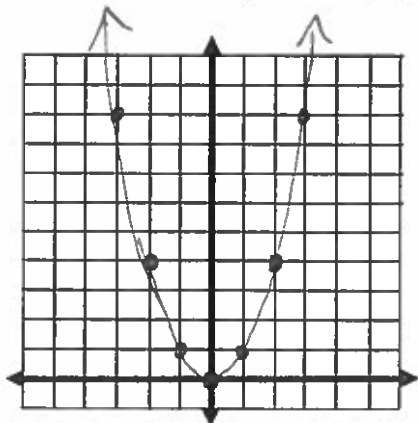
square root each side

The square root function is the inverse of the quadratic function. They are "backwards" of each other, which is why we use one to solve the other.

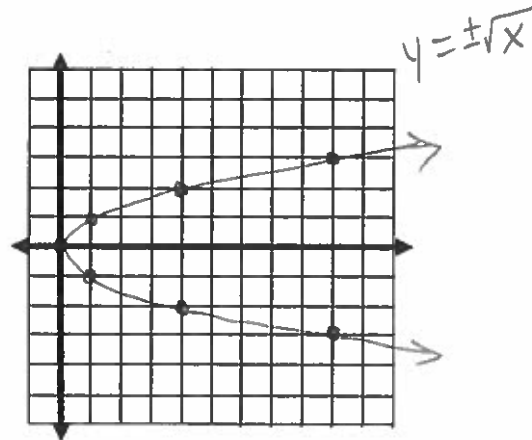
On a graph, inverse functions are created by switching x and y. Let's try this with a quadratic function, to see what a square root function looks like.

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4



x	y
4	-2
1	-1
0	0
1	1
4	2



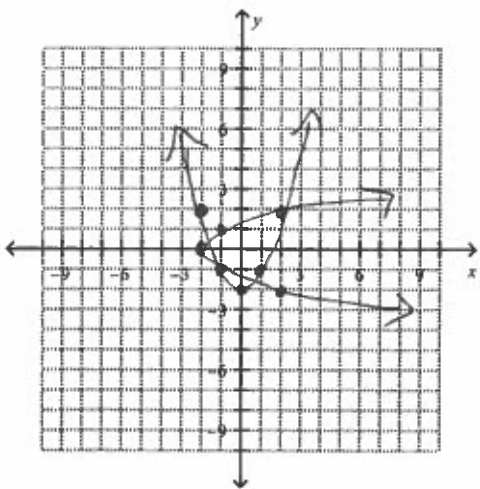
Example 1... Graph the following function and its inverse.

a. $f(x) = x^2 - 2$

x	y	y-2
-2	4	2
-1	1	-1
0	0	-2
1	1	-1
2	4	2

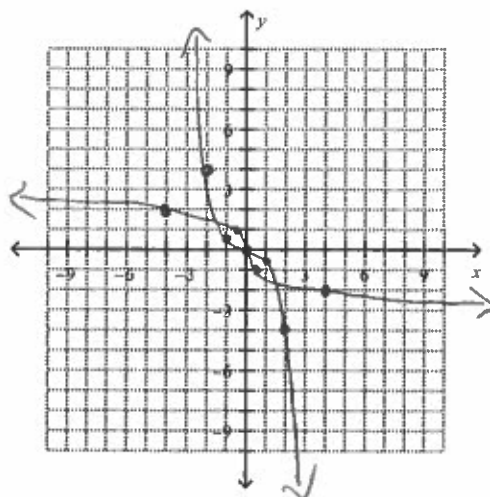
b. $f(x) = -\frac{1}{2}x^3$

x	y	-1/2 y
-2	-4	4
-1	-1/2	1/2
0	0	0
1	1/2	-1/2
2	4	-4



Inverse

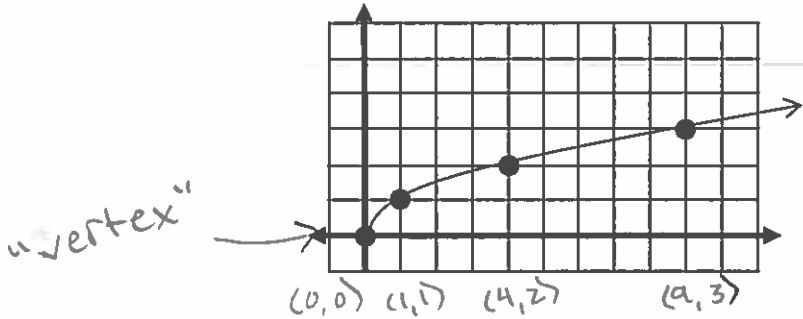
x	y
2	-2
-1	-1
-2	0
-1	1
2	2



Inverse

x	y
4	-2
1/2	-1
0	0
-1/2	1
-4	2

We will graph square root functions using the transformation rules we learned last semester. The parent function of a square root function is $y = \sqrt{x}$ and its graph looks like this:

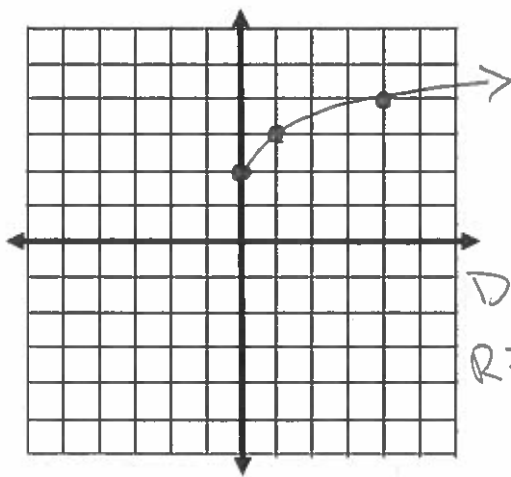


Domain: $[0, \infty)$ or $x \geq 0$

Range: $[0, \infty)$ or $y \geq 0$

Example 2... Graph the radical functions. State the domain and range.

a. $y = \sqrt{x} + 2$

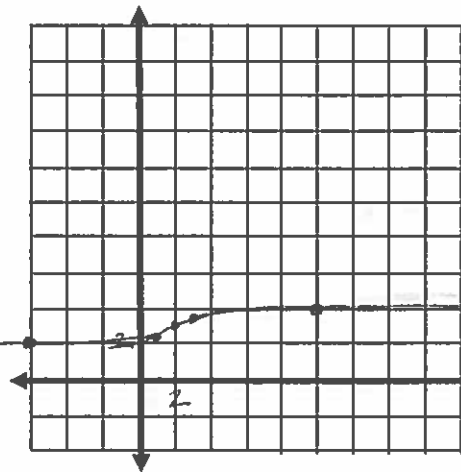


x	y	y+2
0	0	2
1	1	3
4	2	4
9	3	5

D: $[0, \infty)$

R: $[2, \infty)$

b. $y = \frac{1}{2}\sqrt[3]{x-2} + 3$

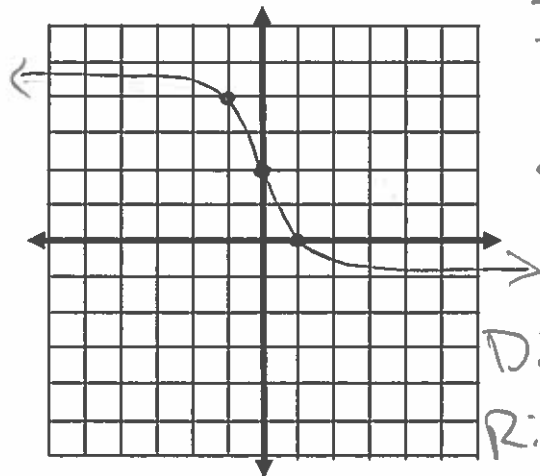


x+2	x	y	1/2 y + 3
-6	-8	-2	2
-1	-1	-1	2.5
0	0	0	3
1	1	1	3.5
2	2	2	4

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

c. $y = -2\sqrt[3]{x} + 2$

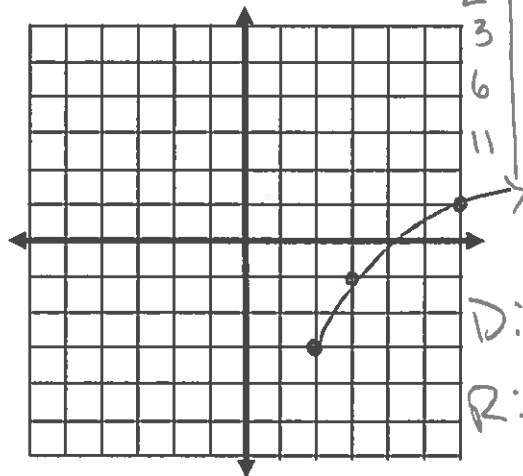


x	y	-2y + 2
-8	-2	6
-1	-1	4
0	0	2
1	1	0
8	2	-2

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

d. $y = 2\sqrt{x-2} - 3$



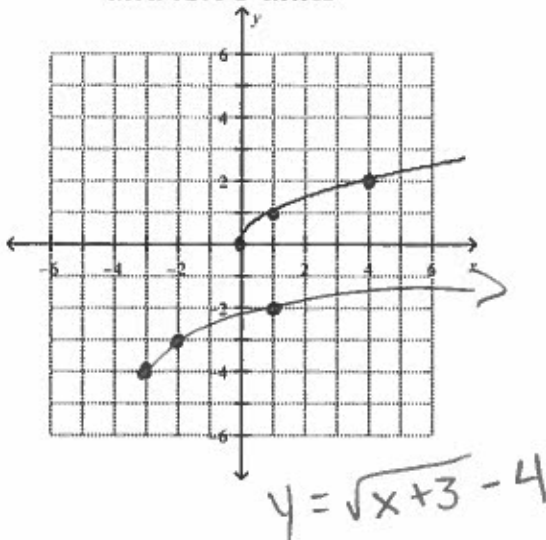
x+2	x	y	2y-3
2	0	0	-3
3	1	1	-1
6	4	2	1
11	9	3	3

D: $[2, \infty)$

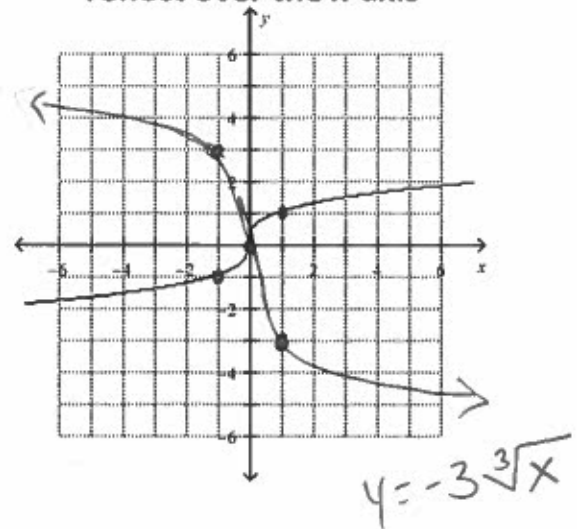
R: $[-3, \infty)$

Example 3... Sketch the graph of the transformation of $f(x) = \sqrt{x}$ and $f(x) = \sqrt[3]{x}$ as described in each exercise. Write the equation to describe each new function.

a. Translate the graph down 4 and left 3 units



b. Stretch vertically by a factor of 3 and reflect over the x-axis



Example 4... Describe how each graph represented by $f(x)$ would be transformed to create the graph represented by $g(x)$.

a. $f(x) = \sqrt{x+2}$

$$g(x) = \sqrt{x+2} + 5$$

up 5

b. $f(x) = \sqrt[3]{x-1}$

$$g(x) = 4\sqrt[3]{x+2} - 6$$

vert. stretch of 4
left + 3
down 6

Example 5... Write an equation for each function by transforming the equation as described.

a. $f(x) = \sqrt{x}$

translated to the right 8 and up 2

$$g(x) = \sqrt{x-8} + 2$$

b. $f(x) = 2\sqrt{x}$

reflected over the y-axis and down 4

$$g(x) = 2\sqrt{-x} - 4$$

c. $f(x) = \sqrt[3]{x}$

translated to the left 1 and stretched vertically by 4

$$g(x) = 4\sqrt[3]{x+1}$$

d. $f(x) = \sqrt[3]{x-1}$

reflected over the x-axis, right 2 and down 3

$$g(x) = -\sqrt[3]{x-3} - 3$$

