Name Notes Date

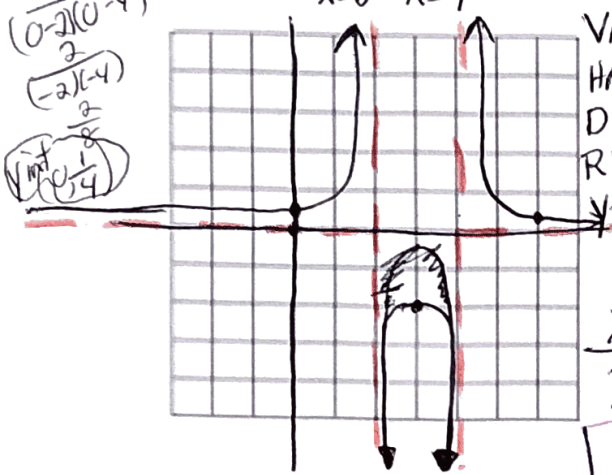
Graphical Discontinuities NOTES (Lesson 2)

Example 1... Sketch each function without using a graphing calculator (you will probably need a small table). Indicate the domain, range, vertical and horizontal asymptote(s), and the y-intercept (when x = 0).

a. $f(x) = \frac{2}{(x-2)(x-4)}$

$\frac{2}{(0-2)(0-4)} = \frac{2}{(-2)(-4)} = \frac{2}{8} = \frac{1}{4}$ (int $(0, \frac{1}{4})$)

X=2 X=4



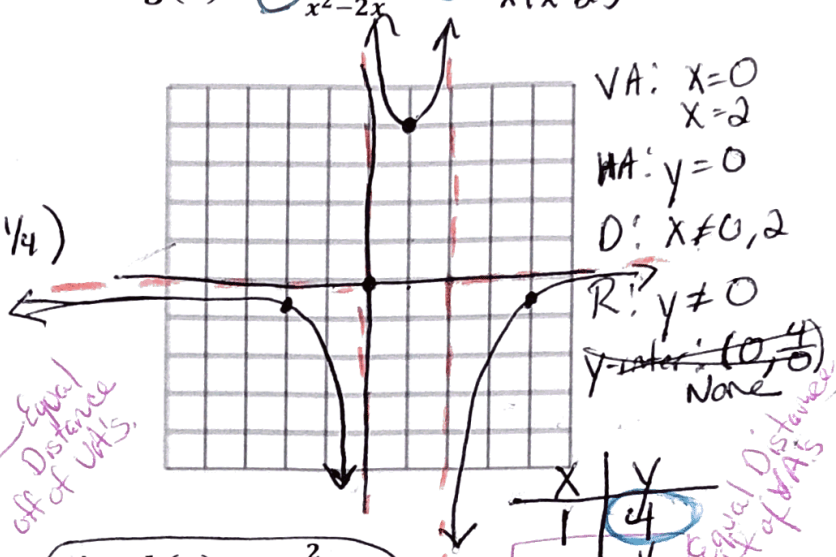
VA: $x=2, x=4$
HA: $y=0$
D: $x \neq 2, 4$
R: $y \neq 0$
y-intercept $(0, 1/4)$

X	Y
3	-2
0	1/4
6	1/4

Equal Distance off of VA's

b. $g(x) = \frac{4}{x^2-2x}$ Compare to example a

$-\frac{4}{x(x-2)}$



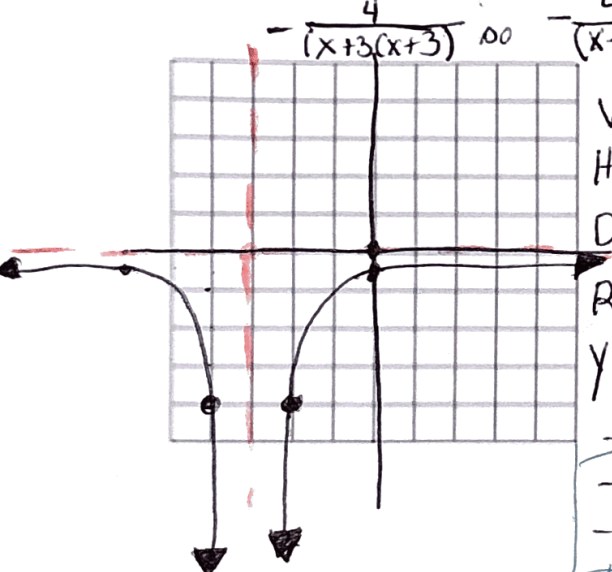
VA: $x=0$, $x=2$
HA: $y=0$
D: $x \neq 0, 2$
R: $y \neq 0$
~~y-inter: (0, 0)~~ None

X	Y
1	-4
-2	-1/2
4	-1/2

Equal Distance off of VA's

c. $p(x) = -\frac{4}{x^2+6x+9}$

$-\frac{4}{(x+3)(x+3)}$ DO $-\frac{4}{(x+3)^2}$



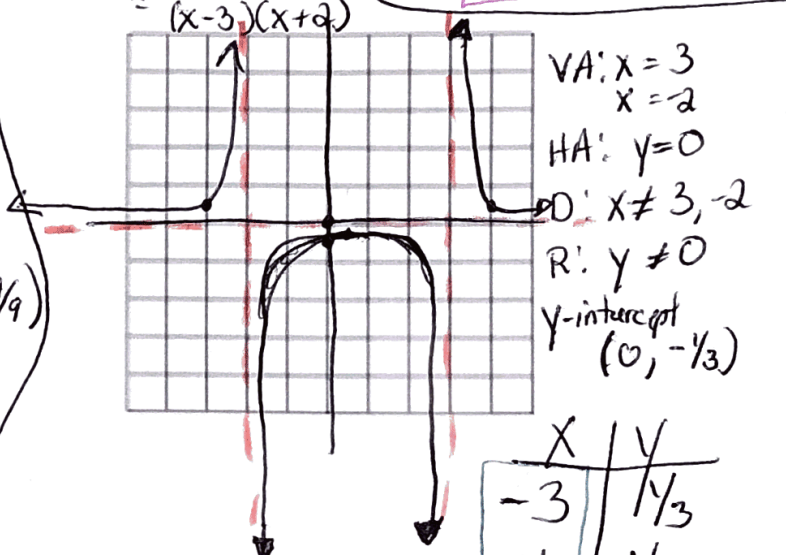
VA: $x=-3$
HA: $y=0$
D: $x \neq -3$
R: $y \neq 0$
y-intercept $(0, -4/9)$

X	Y
-4	-4
-2	-4

1 value off of the VA's

d. $h(x) = \frac{2}{x^2-x-6}$

$= \frac{2}{(x-3)(x+2)}$



VA: $x=3$, $x=-2$
HA: $y=0$
D: $x \neq 3, -2$
R: $y \neq 0$
y-intercept $(0, -1/3)$

X	Y
-3	1/3
4	1/3

These are 4 value off of the VA's

A point of discontinuity is an X-VALUE that would make the denominator zero. (Remember, in math world, really really bad things happen when you divide by zero.)

To FIND a point of discontinuity, you're going to set denominator equal to zero and solve.

Example 2... For each rational function, find any points of discontinuity. (Hint: factor!)

a. $y = \frac{3}{x^2-x-12} = \frac{3}{(x-4)(x+3)}$

Disc: $x = 4, -3$

b. $y = \frac{2x}{3x^2+4}$

Disc: None

$3x^2+4=0$

$\frac{3x^2}{3} = \frac{-4}{3}$

$\sqrt{x^2} = \sqrt{-4/3}$ Not Real

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A point of discontinuity can be one of two different types:

$\frac{(x+2)(x-3)}{(x+2)(x-1)}$ $\frac{(x+2)(x-3)}{(x+2)(x-1)}$

1. A removable discontinuity (hole) – factor cancels out → Hole: $x=-2$
2. A vertical asymptote – factor doesn't cancel out → VA: $x=1$

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A root is where the function crosses the x-axis. In order to get a root(s) the numerator is set equal to zero after the function is simplified. $\frac{(x+2)(x-3)}{(x+2)(x-1)}$ $x-3=0$ $x=3$ (3,0) Root — Must write as a coordinate

Example 3... Describe the vertical asymptotes, removable disc. (holes) and/or roots for the graph of each rational function. (Hint: first, find points of discontinuity. Then decide which kind they are.)

a. $y = \frac{x-2}{(x-1)(x+3)}$ $x-2=0$ $x=2$ so (2,0)

Vertical Asymptotes: $x=1, x=-3$
And/or
Removable Disc.: None
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Roots: (2,0)

b. $y = \frac{x^2-1}{x+1}$ $\frac{(x-1)(x+1)}{(x+1)} = x-1$

Vertical Asymptotes: None
And/or
Removable Disc.: $x=-1$ hole
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Roots: $x-1=0$ (1,0)

c. $y = \frac{x-2}{x^2+x-6}$ $\frac{x-2}{(x-2)(x+3)}$ $\frac{1}{x+3}$

Vertical Asymptotes: $x=-3$
And/or
Removable Disc.: $x=2$ hole
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Roots: None
$1=0$?

d. $y = \frac{x^2+3x}{x+3}$ $\frac{x(x+3)}{(x+3)} = x$

Vertical Asymptotes: None
And/or
Removable Disc.: $x=-3$ hole
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Roots: $x=0$ so (0,0)