Name _____ Date _____

Adding & Subtracting Matrices

A **matrix** is a rectangular array of numbers.

An **element** of a matrix is one of the numbers in the matrix

The **dimensions** of a matrix tell us how many rows and columns it has.

Example 1... Write the dimensions of each matrix and identify the given element.

a. $\begin{bmatrix} 7 & -4 \\ 3 & 12 \end{bmatrix}$; $a_{2,1}$

2x2

$a_{2,1} = 3$

b. $\begin{bmatrix} 2 & 9 & 17 \end{bmatrix}$; $a_{1,2}$

1x3

$a_{1,2} = 9$

c. $\begin{bmatrix} 10 & 0 \\ 2 & -3 \\ 17 & 4 \end{bmatrix}$; $a_{3,2}$

3x2

$a_{3,2} = 4$

When adding or subtracting matrices, the matrices have to have the same size.

Example 2... Find each sum or difference.

a. $\begin{bmatrix} 2 & -4 & 3 \\ 0 & 1 & 12 \end{bmatrix} + \begin{bmatrix} 0 & 7 & -2 \\ 13 & -1 & 5 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 & 1 \\ 13 & 0 & 17 \end{bmatrix}$

b. $2\begin{bmatrix} 6 & 5 \\ 10 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 3 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} 12 & 10 \\ 20 & 0 \end{bmatrix}$

$\begin{bmatrix} 16 & 7 \\ 20 & 1 \end{bmatrix}$

Example 3... Solve each matrix equation, using addition or subtraction.

a. $X - \begin{bmatrix} 2 & 1 \\ 0 & -4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 13 \\ -12 & 0 \end{bmatrix}$ $+\begin{bmatrix} 2 & 1 \\ 0 & -4 \\ 5 & 3 \end{bmatrix}$

$+\begin{bmatrix} & \\ & \\ & \end{bmatrix}$

$\boxed{X = \begin{bmatrix} 7 & 8 \\ 2 & 9 \\ -7 & 3 \end{bmatrix}}$

b. $\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -10 & 2 \end{bmatrix} - X$

$-\begin{bmatrix} 4 & 5 \\ -10 & 2 \end{bmatrix} - \begin{bmatrix} & \\ & \end{bmatrix}$

$\begin{bmatrix} -2 & -4 \\ 10 & -5 \end{bmatrix} = -X$

$\boxed{X = \begin{bmatrix} 2 & 4 \\ -10 & 5 \end{bmatrix}}$

c. $2X + \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ -1 \end{bmatrix}$ $-\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

$-\begin{bmatrix} \\ \\ \end{bmatrix}$

$2X = \begin{bmatrix} 6 \\ -8 \\ -6 \end{bmatrix}$

$\boxed{X = \begin{bmatrix} 3 \\ -4 \\ -3 \end{bmatrix}}$

Example 4... **Find the value of each variable.**

a. $\begin{bmatrix} a & 2b \\ c-2 & d+3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ 10 & 10 \end{bmatrix}$

$a=5$
$b=-3.5$
$c=12$
$d=7$

$2b=-7$
$c-2=10$
$d+3=10$

b. $\begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -13 \\ -10 & -10 \end{bmatrix} = \begin{bmatrix} 2x+1 & -4x \\ 5z & 2.5z-x \end{bmatrix}$

$\begin{bmatrix} 7 & -12 \\ -10 & -8 \end{bmatrix} = \begin{bmatrix} 2x+1 & -4x \\ 5z & 2.5z-x \end{bmatrix}$

$x=3$
$z=-2$

$2x+1=7$
$2x=6$

$5z=-10$

Matrix Multiplication

Example 5... **Mulitply** $3\begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$.

The rules for matrix multiplication are NOT the same as the rules for matrix addition and subtraction.

The Matrix Multiplication Rule: 1st matrix columns equal 2nd matrix rows

WORKS:

$\begin{bmatrix} 2 & -1 & 4 \\ 0 & -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 7 \\ -1 & -5 \\ 2 & 3 \end{bmatrix}$

$2\times3 \cdot 3\times2$

answer $= 2\times2$

DOESN'T WORK:

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} -4 & -3 \\ -1 & -2 \end{bmatrix}$

$2\times3 \times 2\times2$

Example 6... **Multiply each matrix below.**

a. $\begin{bmatrix} 1 & 2 \\ -4 & 3 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 7 & 0 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$3\times2 \quad 2\times3 \qquad 3\times3$

$a=(1)(5)+(2)(-2)=5+-4=1$
$b=(1)(7)+(2)(3)=7+6=13$
$c=(1)(0)+(2)(1)=0+2=2$
$d=(-4)(5)+(3)(-2)=-20+-6=-26$
$e=(-4)(7)+(3)(3)=-28+9=-19$
$f=(-4)(0)+(3)(1)=0+3=3$
$g=(0)(5)+(1)(-2)=0+-2=-2$
$h=(0)(7)+(1)(3)=0+3=3$
$i=(0)(0)+(1)(1)=0+1=1$

$\begin{bmatrix} 1 & 13 & 2 \\ -26 & -19 & 3 \\ -2 & 3 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$2\times2 \quad 2\times2 \qquad 2\times2$

$a=(1)(4)+(0)(1)=4+0=4$
$b=(1)(-2)+(0)(6)=-2+0=-2$
$c=(2)(4)+(1)(1)=8+1=9$
$d=(2)(-2)+(1)(6)=-4+6=2$

$\begin{bmatrix} 4 & -2 \\ 9 & 2 \end{bmatrix}$

To solve a matrix equation $\boldsymbol{AX} = \boldsymbol{B}$, we use the formula $\boldsymbol{X} = \boldsymbol{A}^{-1} \cdot \boldsymbol{B}$

Example 7... **Solve each matrix equation.**

a. $\begin{bmatrix} 0 & 0 & 2 \\ 1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} 6 \\ -11 \\ 8 \end{bmatrix}$

A B

$X = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 1 & -2 \\ 1 & -1 & -9 \end{bmatrix} X = \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix}$

A B

$X = \begin{bmatrix} 4 \\ -10 \\ 1 \end{bmatrix}$

Example 8... **Convert each system to a matrix equation and solve using an inverse matrix & the calculator.**

a. $\begin{cases} x - y - z = -9 \\ 3x + y + 2z = 12 \\ x - y + 2z = 0 \end{cases}$

$\begin{bmatrix} 1 & -1 & -1 \\ 3 & 1 & 2 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ 12 \\ 0 \end{bmatrix}$

coefficient matrix variable matrix constant matrix

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$

b. $\begin{cases} x + y + z = 2 \\ 2x + y = 5 \\ x + 3y - 3z = 14 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 14 \end{bmatrix}$

coeff. var. constant

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.25 \\ 2.5 \\ -1.75 \end{bmatrix}$