

# Polynomials & Linear Factors Notes

Finding the roots (or zeroes or solutions) of a polynomial means to figure out what values of  $x$  would give you  $y = 0$ .

If you remember from Chapter 5, one of the ways to solve an equation is by factoring.

We use the term "multiplicity" when there is more than one of a particular factor.

**Example 1...** Find the zeroes of each function, and state their multiplicity.

a.  $(x-3)^2(x-1)$

$$(x-3)(x-3)(x-1)$$

$$x = 3, 3, 1$$

$$\boxed{x = 3 \text{ mult. } 2}$$

$$\boxed{x = 1}$$

b.  $(x+1)(x-2)(x-3)$

$$\boxed{x = -1, 2, 3}$$

c.  $(x-4)^5(x+2)^3$

$$\boxed{x = 4 \text{ mult. } 5}$$

$$\boxed{x = -2 \text{ mult. } 3}$$

**Example 2...** Write a polynomial function given the following zeroes.

a.  $x = -2, 0, 1$

$$(x+2)(x)(x-1) \leftarrow \text{factors}$$

$$(x^2+2x)(x-1)$$

$$x^3 - x^2 + 2x^2 - 2x$$

$$\boxed{x^3 + x^2 - 2x}$$

b.  $x = -5, -5, 1$

$$(x+5)(x+5)(x-1)$$

$$(x^2+10x+25)(x-1)$$

$$x^3 - x^2 + 10x^2 - 10x + 25x - 25$$

$$\boxed{x^3 + 9x^2 + 15x - 25}$$

**Example 3...** Factor each polynomial completely (\*hint: factor out GCF first!).

a.  $9x^3 + 6x^2 - 3x$

$$3x(3x^2 + 2x - 1)$$

$$\boxed{3x(3x-1)(x+1)}$$

b.  $x^3 + 8x^2 + 16x$

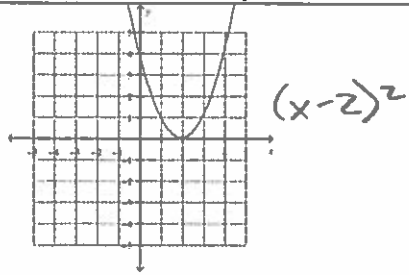
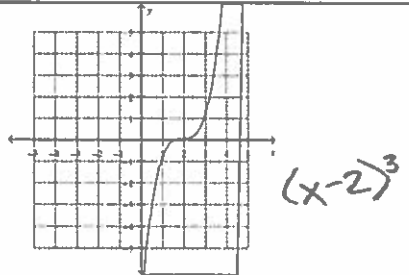
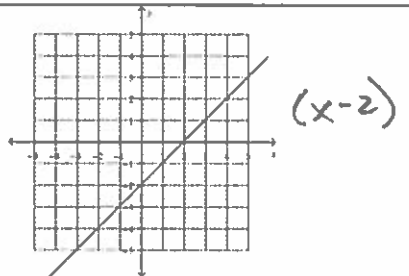
$$x(x^2 + 8x + 16)$$

$$\boxed{x(x+4)(x+4)}$$

or

$$\boxed{x(x+4)^2}$$

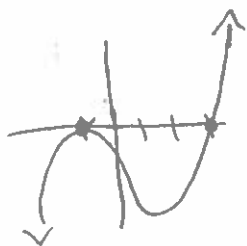
Now that we know how to find the zeroes of a polynomial function, we can combine that knowledge with the end behaviors we learned and use Bump, Wiggle, and Cross to help us sketch a more accurate graph of the polynomial.

For $(x - c)^n$	B, W, C	Example
If $n$ is even	Bump	
If $n$ is odd	Wiggle	
If $n = 1$	Cross	

**Example 4...** Find the zeroes of the function. Then sketch the graph of the function.

a.  $(x - 3)(x + 1)^2$

$x = 3$  cross  
 $x = -1$  mult. 2 bump  
 $x^3$   
 $\downarrow \quad \uparrow$   
 2 turns



b.  $(x + 2)(x - 1)^3(x + 3)^4$

$x = -2$  cross  
 $x = 1$  mult. 3 wiggle  
 $x = -3$  mult. 4 bump  
 $x^8$   
 $\uparrow \quad \uparrow$   
 7 turns

