

Notes: Lesson 5

Rational Exponents

Rational exponents are really just radical expressions in disguise. Use the following rule to convert between the two:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \sqrt[n]{a}^m$$

Example 1... Convert to radical form/exponent form (whichever it's not already in).

a. $x^{2/7}$

$$\sqrt[7]{x^2} \text{ or } \sqrt{x^2}$$

b. $\sqrt[4]{c^3}$

$$c^{3/4}$$

c. $4^{1/2}$

$$\sqrt{4} = 2$$

Properties of Rational Exponents	
Property:	Example:
$x^m \cdot x^n = x^{m+n}$	$x^2 \cdot x^5 = x^7$
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^6}{x^2} = x^4$
$(x^m)^n = x^{m \cdot n}$	$(y^3)^4 = y^{12}$
$(x \cdot y)^m = x^m \cdot y^m$	$(ab)^3 = a^3 b^3$
$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$
$x^{-m} = \frac{1}{x^m} \quad / \quad \frac{1}{x^{-m}} = x^m$	$2a^{-3} = \frac{2}{a^3} \quad / \quad \frac{4}{y^{-5}} = 4y^5$

Example 2... Use properties of exponents to simplify each expression as much as possible.

a. $64^{1/3}$

$$\sqrt[3]{64} = \boxed{4}$$

b. $7^{1/2} \cdot 7^{1/2}$

$$\sqrt{7} \cdot \sqrt{7} = \sqrt{49} = \boxed{7}$$

c. $5^{1/3} \cdot 25^{1/3}$

$$\sqrt[3]{5} \cdot \sqrt[3]{25}$$

$$\sqrt[3]{125} = \boxed{5}$$

d. $32^{3/5}$

$$\sqrt[5]{32^3} = 2^3 = \boxed{8}$$

e. $(3x^{2/3})^{-3}$ $\frac{2}{3} \cdot \frac{-3}{1} = \frac{-6}{3} = -2$

$$3^{-3} x^{-2}$$

$$\frac{1}{3^3 x^2} = \boxed{\frac{1}{27x^2}}$$

f. $(x^{2/3} y^{-1/6})^{-12}$ $\frac{2}{3} \cdot \frac{-12}{1} = \frac{-24}{3} = -8$

$$x^{-8} y^2$$

$$-\frac{1}{6} \cdot \frac{-12}{1} = \frac{12}{6} = 2$$

$$\boxed{\frac{y^2}{x^8}}$$

g. Write the expression in simplest form: $(8x^{15})^{-1/3}$

$$8^{-1/3} x^{-5}$$

$$\frac{15}{1} \cdot \frac{-1}{3} = \frac{-15}{3} = -5$$

$$\frac{1}{8^{1/3} x^5} = \frac{1}{\sqrt[3]{8} x^5} = \boxed{\frac{1}{2x^5}}$$