Name _____ Date _____

Solving Polynomials by Factoring Notes

There are a couple different ways to factor cubic functions. One of them (grouping) works for one type of special case and the other (using a formula) works for other special cases. We will look at both of them in this lesson.

Sum and Difference of Cubes: This method works when you have a perfect cube for both terms.

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Quad. Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1... Factor the expression using the correct formula and then solve.

a. $\overset{a^3}{x^3} + \overset{b^3}{64} = 0$ $\quad a = x$, $b = 4$

$(x+4)((x)^2 - (x)(4) + (4)^2) = 0$

$(x+4)(x^2 - 4x + 16) = 0$

$x + 4 = 0$

$\boxed{x = -4 \quad x = 2 \pm 2i\sqrt{3}}$

$\frac{-4 \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$

$\frac{4 \pm \sqrt{-48}}{2}$ $\quad -48 < 16$, 3

$= \frac{4 \pm 4i\sqrt{3}}{2}$

b. $\overset{a^3}{x^3} - \overset{b^3}{27} = 0$ $\quad a = x$, $b = 3$

$(x-3)((x)^2 + (x)(3) + (3)^2) = 0$

$(x-3)(x^2 + 3x + 9) = 0$

$\boxed{x = 3 \quad x = \frac{-3 \pm 3i\sqrt{3}}{2}}$

$\frac{-3 \pm \sqrt{(3)^2 - 4(1)(9)}}{2(1)}$

$\frac{-3 \pm \sqrt{-27}}{2}$ $\quad -27 < 9$, 3

$\frac{-3 \pm 3i\sqrt{3}}{2}$

c. $8x^3 - 64 = 0$ $\quad 8$

$x^3 - 8 = 0$ $\quad a = x$, $b = 2$

$(x-2)(x^2 + 2x + 4) = 0$

$\boxed{x = 2 \quad x = -1 \pm i\sqrt{3}}$

$\frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$

$\frac{-2 \pm \sqrt{-12}}{2}$ $\quad -12 < 4$, 3

$\frac{-2 \pm 2i\sqrt{3}}{2}$

d. $27x^3 + 1 = 0$ $\quad a = 3x$, $b = 1$

$(3x+1)((3x)^2 + (3x)(1) + (1)^2) = 0$

$(3x+1)(9x^2 + 3x + 1) = 0$

$\boxed{x = -\frac{1}{3} \quad x = \frac{-1 \pm i\sqrt{3}}{6}}$

$\frac{-3 \pm \sqrt{(3)^2 - 4(9)(1)}}{2(9)}$

$\frac{-3 \pm \sqrt{-27}}{18}$ $\quad -27 < 9$, 3

$\frac{-3 \pm 3i\sqrt{3}}{18}$

Example 2... Factor the expression using grouping and then solve.

a. $(x^3 + 3x^2)(-4x - 12) = 0$

$x^2(x+3) - 4(x+3) = 0$

$(x+3)(x^2-4) = 0$

$(x+3)(x+2)(x-2) = 0$

$\boxed{x = -3, -2, 2}$

b. $(x^3 - 2x^2)(+2x - 4) = 0$

$x^2(x-2) + 2(x-2) = 0$

$(x-2)(x^2+2) = 0$

$\boxed{x = 2}$

$x^2 + 2 = 0$

$x^2 = -2$

$\boxed{x = \pm i\sqrt{2}}$

c. $(2x^3 + x^2)(+6x + 3) = 0$

$x^2(2x+1) + 3(2x+1) = 0$

$(2x+1)(x^2+3) = 0$

$\boxed{x = -\frac{1}{2}}$

$x^2 + 3 = 0$

$x^2 = -3$

$\boxed{x = \pm i\sqrt{3}}$

d. $(6x^3 - 8x^2)(-9x + 12) = 0$

$2x^2(3x-4) - 3(3x-4) = 0$

$(3x-4)(2x^2-3) = 0$

$\boxed{x = \frac{4}{3}}$

$\boxed{x = \pm\frac{\sqrt{6}}{2}}$

$2x^2 - 3 = 0$

$x^2 = \frac{3}{2}$

$x = \pm\frac{\sqrt{3}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{6}}{2}$

Example 3... The final method for factoring and solving cubics is to find a greatest common factor first and then solve.

a. $x^3 + 3x^2 - 4x = 0$

$x(x^2 + 3x - 4) = 0$

$x(x+4)(x-1) = 0$

$\boxed{x = 0, -4, 1}$

b. $2x^3 - 6x^2 + 4x = 0$

$2x(x^2 - 3x + 2) = 0$

$2x(x-2)(x-1) = 0$

$\boxed{x = 0, 2, 1}$

c. $x^3 + 2x^2 + 10x = 0$

$x(x^2 + 2x + 10)$

$\boxed{x = 0}$

$\boxed{x = -1 \pm 3i}$

$\frac{-2 \pm \sqrt{(2)^2 - 4(1)(10)}}{2(1)}$

$\frac{-2 \pm \sqrt{-36}}{2}$

$\frac{-2 \pm 6i}{2}$

d. $2x^3 - 8x^2 + 14x = 0$

$2x(x^2 - 4x + 7) = 0$

$\boxed{x = 0}$

$\boxed{x = 2 \pm i\sqrt{3}}$

$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$

$\frac{4 \pm \sqrt{-12}}{2}$ $< \frac{4}{3}$

$\frac{4 \pm 2i\sqrt{3}}{2}$

We can also solve quartic functions that use a quadratic pattern by factoring.

Example 4... Factor each polynomial.

a. $x^4 - 8x^2 + 12$

$(x^2-6)(x^2-2)$

b. $2x^4 + 9x^2 - 5$

$(2x^2-1)(x^2+5)$

Example 5... Factor and solve each polynomial equation.

a. $x^4 + 8x^2 - 9 = 0$

$(x^2+9)(x^2-1)=0$

$(x^2+9)(x+1)(x-1)=0$

$x^2+9=0$

$x^2=-9$

$\boxed{x=\pm 3i, -1, 1}$

b. $3x^4 - 11x^2 + 6 = 0$

$(3x^2-2)(x^2-3)=0$

$3x^2-2=0$

$x^2=2/3$

$x=\pm\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$x^2-3=0$

$x^2=3$

$\boxed{x=\pm\frac{\sqrt{6}}{3}, \pm\sqrt{3}}$

Example 6... Solve each equation by using one of the factoring methods in the other examples – remember to look for GCF!

a. $2x^4 + 18x^3 = 0$

$2x^3(x+9)=0$

$\boxed{x=0 \text{ mult. } 3 \quad x=-9}$

b. $x^3 = 4x^2 + 12x$

$x^3-4x^2-12x=0$

$x(x^2-4x-12)=0$

$x(x-6)(x+2)=0$

$\boxed{x=0, 6, -2}$