Name _____ Date _____

Solving Radical Equations

Notes: Lesson 6

A radical equation is an equation that has a variable inside of a radical, or a variable with a rational exponent.

Example 1... Solve the following equations. Check your solution.

a. $2+\sqrt{3x-2}=6$

$-2 \quad -2$

$(\sqrt{3x-2})^2=(4)^2$

$3x-2=16$

$+2 \quad +2$

$\frac{3x}{3}=\frac{18}{3}$

$\boxed{x=6}$

Check:

$2+\sqrt{3(6)-2}=6$

$2+\sqrt{18-2}=6$

$2+\sqrt{16}=6$

$2+4=6$

$6=6$ ✓

b. $3\sqrt{5x+1}-6=0$

$+6 \quad +6$

$\frac{3\sqrt{5x+1}}{3}=\frac{6}{3}$

$(\sqrt{5x+1})^2=(2)^2$

$5x+1=4$

$-1 \quad -1$

$\frac{5x}{5}=\frac{3}{5}$

$\boxed{x=3/5}$

Check:

$3\sqrt{5(3/5)+1}-6=0$

$3\sqrt{3+1}-6=0$

$3\sqrt{4}-6=0$

$3(2)-6=0$

$6-6=0$

$0=0$ ✓

c. $\frac{2(x-2)^{2/3}}{2}=\frac{50}{2}$

$(x-2)^{2/3\cdot 3/2}=25^{3/2}$

$x-2=\sqrt{25}^3$

$x-2=\pm 5^3$

$x-2=\pm 125$

$x-2=125$ $\quad$ $x-2=-125$

$\boxed{x=127}$ $\quad$ $\boxed{x=-123}$

check:

$2(127-2)^{2/3}=50$

$2(125)^{2/3}=50$

$2\sqrt[3]{125}^2=50$

$2(5)^2=50$

$2(25)=50$ ✓

check:

$2(-123-2)^{2/3}=50$

$2(-125)^{2/3}=50$

$2\sqrt[3]{-125}^2=50$

$2(-5)^2=50$

$2(25)=50$ ✓

d. $3(x+3)^{3/2}-5=76$

$+5 \quad +5$

$\frac{3(x+3)^{3/2}}{3}=\frac{81}{3}$

$(x+3)^{3/2\cdot 2/3}=27^{2/3}$

$x+3=\sqrt[3]{27}^2$

$x+3=3^2$

$x+3=9$

$\boxed{x=6}$

Check:

$3(6+3)^{3/2}-5=76$

$3(9)^{3/2}-5=76$

$3\sqrt{9}^3-5=76$

$3(3)^3-5=76$

$3(27)-5=76$

$81-5=76$ ✓

Example 2... Solve the equation, and check for extraneous solutions.

a. $\sqrt{3x+2} - \sqrt{2x+7} = 0$

$+\sqrt{2x+7}$ $+\sqrt{2x+7}$

$(\sqrt{3x+2})^2 = (\sqrt{2x+7})^2$

$3x+2 = 2x+7$

$-2x \quad -2x$

$x+2 = 7$

$-2 \quad -2$

$\boxed{x=5}$

Check:

$\sqrt{3(5)+2} - \sqrt{2(5)+7} = 0$

$\sqrt{15+2} - \sqrt{10+7} = 0$

$\sqrt{17} - \sqrt{17} = 0$

$0 = 0$ ✓

b. $(2x+1)^{0.5} - (3x+4)^{0.5} = 0$

$+(3x+4)^{.5}$ $+(3x+4)^{.5}$

$(2x+1)^{.5} = (3x+4)^{.5}$

$(2x+1)^{1/2 \cdot 2/1} = (3x+4)^{1/2 \cdot 2/1}$

$2x+1 = 3x+4$

$-2x \quad -2x$

$1 = x+4$

$-4 \quad -4$

$\boxed{x=-3}$

Check:

$(2(-3)+1)^{.5} - (3(-3)+4)^{.5} =$

$(-6+1)^{.5} - (-9+4)^{.5} = 0$

$(-5)^{.5} - (-5)^{.5} = 0$

$0 = 0$

c. $\sqrt{x+7} - x = 1$

$+x \quad +x$

$\sqrt{x+7}^2 = (x+1)^2$

$x+7 = x^2+2x+1$

$-x \ -7 \quad -x \ -7$

$0 = x^2+x-6$

$0 = (x+3)(x-2)$

$\boxed{x=-3}$ extran. $\quad \boxed{x=2}$

Check:

$\sqrt{-3+7} - (-3) = 1$

$\sqrt{4} + 3 = 1$

$2+3 = 1$

$5 = 1$ ✗

Check:

$\sqrt{2+7} - 2 = 1$

$\sqrt{9} - 2 = 1$

$3-2 = 1$

$1 = 1$ ✓

d. $\sqrt{3x-2}^2 = (x-2)^2$

$3x-2 = x^2-4x+4$

$-3x \ +2 \quad -3x \ +2$

$0 = x^2-7x+6$

$0 = (x-1)(x-6)$

$\boxed{x=1}$ extran. $\quad \boxed{x=6}$

Check:

$\sqrt{3(1)-2} = 1-2$

$\sqrt{3-2} = -1$

$\sqrt{1} = -1$

$1 = -1$ ✗

Check:

$\sqrt{3(6)-2} = 6-2$

$\sqrt{18-2} = 4$

$\sqrt{16} = 4$

$4 = 4$ ✓